International Journal of Applied Mathematics
& Statistical Sciences (IJAMSS)
ISSN(P):2319-3972; ISSN(E): 2319-3980
Vol. 3, Issue 6, Nov 2014, 1-8

International Academy of Science, Engineering and Technology
Connecting Researchers; Nurturing Innovations

COMPARATIVE STUDY OF STANDBY COMPRESSOR SYSTEMS WITH AND WITHOUT PROVISION OF PRIORITY TO FAILED COMPRESSOR UNIT

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ABSTRACT

The present paper is an attempt to compare two standby systems consisting of two compressor units where one compressor is in operative state and other is in standby state at initial stage. In Model 1 priority is given to failed compressor unit whereas in Model 2 there is no concept of priority. Any major failure or annual maintenance brings the operating unit to a complete halt. It has been observed that the unit can fail due to various types of failures which can be categorized as serviceable type, repairable type and replaceable type. For availability analysis of the unit real failure as well as repair time data from a milk plant have been collected and measures of unit effectiveness i.e. availability and mean time to unit failure for both the models has been computed graphically as well as numerically by using semi-Markov process and regenerative point technique.

KEYWORDS: Compressor Unit, Regenerative Point Technique, Refrigeration System, Semi-Markov Process

INTRODUCTION

In a field of reliability standby systems have been discussed by various researchers including [1-4] under various assumptions/considerations. For graphical study, they have taken assumed values for failure and repair rates, and not used the observed values. However, some researchers including [5-8] studied some reliability models collecting real data on failure and repair rates of the units used in systems.

A potential application of the reliability concepts has been recently explored in terms profit analysis of a two unit standby oil delivery system with off line repair facility when priority is given to partially failed unit over the completely failed unit for repair and system having a provision of switching over to another system and thereby achieving some reliability measures of the delivering system effectiveness which in turn are meaningful in understanding the profit analysis of such system by Rekha Narang and Upasana Sharma .

Getting inspiration from the above concept the present paper is thus a attempt to, to compare two standby systems consisting of two compressor units where one compressor is in operative state and other is in standby state at initial stage. In present paper two unit standby model are developed by considering the real failure situations as depicted in the data for analysis. For this purpose, a refrigeration system used in milk plant is identified. In milk plant's refrigeration system compressor plays an important role. Any major failure or annual maintenance brings the operating unit to a complete halt. It has been observed that the unit can fail due to various types of failures which can be categorized as- serviceable type, repairable type and replaceable type. Initially one unit is operative and the other is standby .On the failure of operative unit ,it can be serviced ,repaired and replaced depending upon type of failure category.

For availability analysis of the unit real failure as well as repair time data from a milk plant have been collected and measures of unit effectiveness i.e. availability and mean time to unit failure has been computed graphically as well as numerically by using semi-Markov process and regenerative point technique.

Abbreviations and Acronyms

O Operative Unit

$\lambda_{11},\lambda_{21}$ Failure rate when failure is of serviceable type for first and second compressor respectively

$\lambda_{12},\lambda_{22}$ Failure rate when failure is of repairable type for first and second compressor respectively

$\lambda_{13},\lambda_{23}$ Failure rate when failure is of replaceable type for first and second compressor respectively

$\alpha_{11},\alpha_{12},\alpha_{13}$ Repair rates when failure is of serviceable, repairable and replaceable type for first compressor.

$G_{11}(t),g_{11}(t)$, $G_{21}(t),g_{21}(t)$ c. d. f and p. d. f of time for service when failure is of serviceable type for first compressor and second compressor respectively

$G_{12}(t),g_{12}(t)$, $G_{22}(t),g_{22}(t)$ c. d. f and p. d. f. of time for repair when failure is of repairable type for first compressor and second compressor respectively

$G_{13}(t),g_{13}(t)$, $G_{23}(t),g_{23}(t)$ c. d. f and p. d. f of time for replacement when failure is of replaceable type for first compressor and second compressor respectively

Q_{ij} (t) cumulative distribution function (c. d. f) of first passage time from a regenerative state i to j or to a failed state j in (0, t].

$ø_i$ (t) c. d. f of first passage time from a regenerate state i to a failed state j.

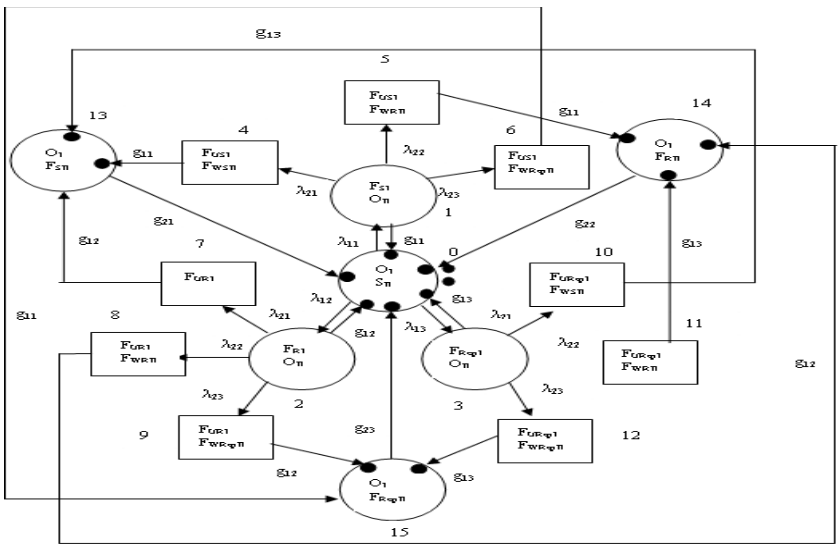


Figure 1: State Transition Diagram (Model 1)

Transition Probabilities and Mean Sojourn Times

A state transition diagram showing the various states of transition of the system is shown in Figure 1. The epochs of entry into states 0,1,2,3 are regenerative states. States 4, 5, 6,7, 8, 9,10,11,12 are down states. The non zero elements p_{ij} are given below:

$$p_{01}=\frac{\lambda_{11}}{\lambda^*},\ p_{02}=\frac{\lambda_{12}}{\lambda^*},\ p_{03}=\frac{\lambda_{13}}{\lambda^*}\ where\ \lambda^*=\lambda_{11}+\lambda_{12}+\lambda_{13},\ p_{10}=g_{11}^*(\lambda),\ p_{20}=g_{12}^*(\lambda),\ p_{30}=g_{13}^*(\lambda)\ where\ \lambda=\lambda_{21}+\lambda_{22}+\lambda_{23}$$

$$p_{27},p_{2,13}^{7}=\frac{\lambda_{21}}{\lambda}(1-g_{12}^*(\lambda));\ p_{28},p_{2,14}^{8}=\frac{\lambda_{22}}{\lambda}(1-g_{12}^*(\lambda));\ p_{29},p_{2,15}^{9}=\frac{\lambda_{23}}{\lambda}(1-g_{12}^*(\lambda)),\ p_{14},p_{1,13}^{4}=\frac{\lambda_{21}}{\lambda}(1-g_{11}^*(\lambda));\ p_{15},p_{1,14}^{5}=\frac{\lambda_{22}}{\lambda}(1-g_{11}^*(\lambda))$$

$$p_{16},p_{1,15}^{6}=\frac{\lambda_{23}}{\lambda}(1-g_{11}^*(\lambda)),\ p_{3,10},p_{3,13}^{10}=\frac{\lambda_{21}}{\lambda}(1-g_{13}^*(\lambda));\ p_{3,11},p_{3,14}^{11}=\frac{\lambda_{22}}{\lambda}(1-g_{13}^*(\lambda));\ p_{3,12},p_{3,15}^{12}=\frac{\lambda_{23}}{\lambda}(1-g_{13}^*(\lambda)),\ p_{10}+p_{14}+p_{15}+p_{16}=1,$$

$$p_{10}+p_{1,13}^{4}+p_{1,14}^{5}+p_{1,15}^{6}=1,\ p_{20}+p_{27}+p_{28}+p_{29}=1,\ p_{20}+p_{2,13}^{7}+p_{2,14}^{8}+p_{2,15}^{9}=1,\ p_{30}+p_{3,10}+p_{3,11}+p_{3,12}=1,\ p_{30}+p_{3,13}^{10}+p_{3,14}^{11}+p_{3,15}^{12}=1$$

The mean sojourn time (μ_i) in the regenerative state 'i' is defined as time of stay in that state before transition to any other state:

$$\mu_0=\frac{1}{\lambda_{11}+\lambda_{12}+\lambda_{13}},\ \mu_1=\frac{1}{\lambda_{21}+\lambda_{22}+\lambda_{23}},\ \mu_2=\frac{1}{\lambda_{21}+\lambda_{22}+\lambda_{23}},\ \mu_3=\frac{1}{\lambda_{21}+\lambda_{22}+\lambda_{23}},\ \mu_4=\int_0^\infty \overline{G}_{21}(t)dt,\ \mu_5=\int_0^\infty \overline{G}_{22}(t)dt$$

$$\mu_6=\int_0^\infty \overline{G}_{23}(t)dt,\ \mu_7=\int_0^\infty \overline{G}_{21}(t)dt,\ \mu_8=\int_0^\infty \overline{G}_{22}(t)dt,\ \mu_9=\int_0^\infty \overline{G}_{23}(t)dt,\ \mu_{10}=\int_0^\infty \overline{G}_{21}(t)dt,\ \mu_{11}=\int_0^\infty \overline{G}_{22}(t)dt,\ \mu_{12}=\int_0^\infty \overline{G}_{23}(t)dt$$

The unconditional mean time taken by the system to transit for any regenerative state 'j' when it (time) is counted from the epoch of entrance into state 'i' is mathematically state as:

$$mij=\int_0^\infty t\,dQ_{ij}(t)=-q_{ij}'^*(0)$$

$$m_{01}+m_{02}+m_{03}=\frac{1}{(\lambda^*)}=\mu_0,\ m_{10}+m_{14}+m_{15}+m_{16}=\mu_1(1-g_{11}^*(\lambda),\ m_{20}+m_{27}+m_{28}+m_{29}=\mu_2(1-g_{12}^*(\lambda),\ m_{30}+m_{3,10}+m_{3,11}+m_{3,12}=\mu_3(1-g_{13}^*(\lambda)$$

Mean Time to System Failure

To determine the mean time to system failure (MTSF) of the system, we regard the failed states of the system as absorbing states. Now mean time to system failure (MTSF) when unit started at the beginning of state 0 is

$$\text{MTSF}=T_0=\lim_{S\to O}\frac{1-\varnothing_0^{**}(s)}{s}=\frac{N}{D}$$

$$\varnothing_0^{**}(s)=\frac{N(s)}{D(s)}$$

$$\text{Where } N(s)=\varnothing_{03}^{**}(s)\varnothing_{3,10}^{**}(s)+\varnothing_{03}^{**}(s)\varnothing_{3,11}^{**}(s)+\varnothing_{03}^{**}(s)\varnothing_{3,12}^{**}(s)+\varnothing_{01}^{**}(s)\varnothing_{14}^{**}(s)+\varnothing_{01}^{**}(s)\varnothing_{15}^{**}(s)+\varnothing_{01}^{**}(s)\varnothing_{16}^{**}(s)+\varnothing_{02}^{**}(s)\varnothing_{27}^{**}(s)$$
$$+\varnothing_{02}^{**}(s)\varnothing_{28}^{**}(s)+\varnothing_{02}^{**}(s)\varnothing_{29}^{**}(s)$$

$$D(s)=1-\varnothing_{01}^{**}(s)\varnothing_{10}^{**}(s)-\varnothing_{02}^{**}(s)\varnothing_{20}^{**}(s)-\varnothing_{03}^{**}(s)\varnothing_{30}^{**}(s)$$

where $N=8907.480982, D=0.6391011951$

MTSF=13973.51263 Hrs

Availability Analysis

Let $A_i(t)$ be the probability that the system is in upstate at instant t given that the system entered regenerative state i at t=0. In steady state availability of the system is given by

$$A_0 = \lim_{s \to 0}(sA_0^*(s)) = N_1 / D_1$$

where

$$N_1 = \mu_0, D_1 = \mu_0 + p_{01}\mu_1(1-g_{11}^*(\lambda)) + p_{02}(\mu_2(1-g_{12}^*(\lambda)) + (p_{03}\mu_3(1-g_{13}^*(\lambda)) + m_{13,0}(p_{01}p_{1,13}^4 + p_{02}p_{2,13}^7 + p_{03}p_{3,13}^{10}) + m_{14,0}(p_{01}p_{1,14}^5 + p_{02}p_{2,14}^8 + p_{03}p_{3,14}^{11})$$
$$+ m_{15,0}(p_{01}p_{1,15}^6 + p_{02}p_{2,15}^9 + p_{03}p_{3,15}^{12})$$

N_1 =8617.718028, D_1=13998.305227407

Availability =.6156

Model 2

Model Description and Assumptions

- The unit is initially operative at state 0 and its transition depends upon the type of failure category to any of the three states 1 to 3 with different failure rates.
- All failure times are assumed to have exponential distribution
- After each servicing/ repair/replacement at states the unit works as good as new.
- Priority given to failed unit for service, repair and replacement.

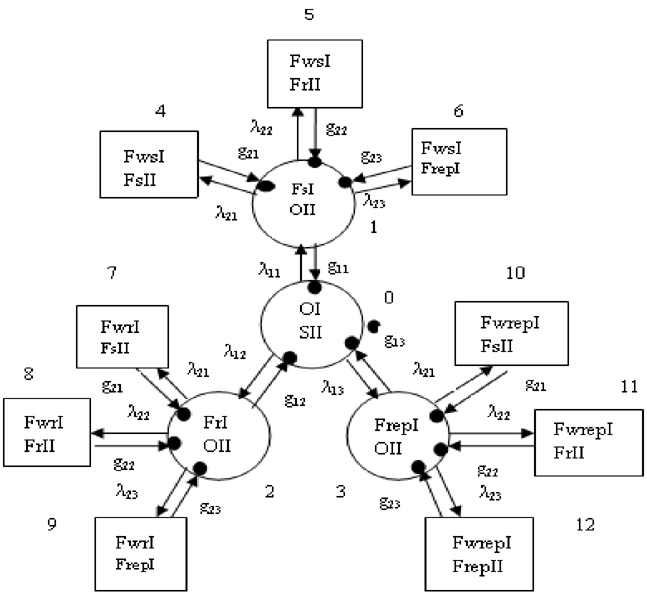


Figure 2: Transition State Diagram (Model2)

Transition Probabilities and Mean Sojourn Times

A state transition diagram showing the various states of transition of the system is shown in Figure 1. The epochs of entry into states 0,1,2,3 are regenerative states. States 4, 5, 6,7,8,9,10,11,12 are down states. The non zero elements p_{ij} are given below The mean sojourn time (μ_i) in the regenerative state 'i' is defined as time of stay in that state before transition to any other state:

$$p_{01} = \frac{\lambda_{11}}{\lambda^*}, p_{02} = \frac{\lambda_{12}}{\lambda^*}, p_{03} = \frac{\lambda_{13}}{\lambda^*}, p_{10} = g_{11}^*(\lambda), p_{20} = g_{12}^*(\lambda), p_{30} = g_{13}^*(\lambda), p_{27} = \frac{\lambda_{21}}{\lambda}(1-g_{12}^*(\lambda)), where \lambda^* = \lambda_{11} + \lambda_{12} + \lambda_{13}, \lambda = \lambda_{21} + \lambda_{22} + \lambda_{23}$$

$$p_{28} = \frac{\lambda_{22}}{\lambda}(1-g_{12}^*(\lambda)), p_{29} = \frac{\lambda_{23}}{\lambda}(1-g_{12}^*(\lambda)) p_{14} = \frac{\lambda_{21}}{\lambda}(1-g_{11}^*(\lambda)), p_{15} = \frac{\lambda_{22}}{\lambda}(1-g_{11}^*(\lambda)) p_{16} = \frac{\lambda_{23}}{\lambda}(1-g_{11}^*(\lambda)), p_{3,10} = \frac{\lambda_{21}}{\lambda}(1-g_{13}^*(\lambda))$$

$$p_{3,11}=\frac{\lambda_{22}}{\lambda}(1-g_{13}^{*}(\lambda))$$

$$\mu_0=\frac{1}{\lambda^*},\mu_1=\frac{1}{\lambda},\mu_2=\frac{1}{\lambda},\mu_3=\frac{1}{\lambda},\mu_4=\int_0^\infty \overline{G}_{21}(t)dt,\mu_5=\int_0^\infty \overline{G}_{22}(t)dt,\mu_6=\int_0^\infty \overline{G}_{23}(t)dt\mu_7=\int_0^\infty \overline{G}_{21}(t)dt,\mu_8=\int_0^\infty \overline{G}_{22}(t)dt$$

$$\mu_9=\int_0^\infty \overline{G}_{23}(t)dt,\mu_{10}=\int_0^\infty \overline{G}_{21}(t)dt,\mu_{11}=\int_0^\infty \overline{G}_{22}(t)dt,\mu_{12}=\int_0^\infty \overline{G}_{23}(t)dt$$

The unconditional mean time taken by the system to transit for any regenerative state 'j' when it (time) is counted from the epoch of entrance into state 'i' is mathematically stated as:

$$mij=\int_0^\infty tdQ_{ij}(t)=-q_{ij}'^*(0),m_{01}+m_{02}+m_{03}=\frac{1}{(\lambda^*)}=\mu_0,m_{10}+m_{11}^4+m_{11}^5+m_{11}^6=\mu_1(1-g_{11}^*(\lambda),m_{20}+m_{22}^7+m_{22}^8+m_{22}^9=\mu_2(1-g_{12}^*(\lambda),$$

$$m_{20}+m_{27}+m_{28}+m_{29}=\mu_2(1-g_{12}^*(\lambda),m_{30}+m_{33}^{10}+m_{33}^{11}+m_{33}^{12}=\mu_3(1-g_{13}^*(\lambda),m_{30}+m_{3,10}+m_{3,11}+m_{3,12}=\mu_3(1-g_{13}^*(\lambda)$$

Mean Time to System Failure

To determine the mean time to system failure (MTSF) of the system, we regard the failed states of the system absorbing. Now mean time to system failure (MTSF) when unit started at the beginning of state

$$\text{MTSF}=\text{T}_0=\lim_{s\to 0}\frac{1-\text{Ø}_0{}^{**}(s)}{\text{s}}=\frac{N}{D}$$

$$\text{Ø}_0{}^{**}(s)=\frac{N(s)}{D(s)}$$

$$\text{Where N(s)}=\text{Ø}_{03}{}^{**}(s)\text{Ø}_{3,10}{}^{**}(s)+\text{Ø}_{03}{}^{**}(s)\text{Ø}_{3,11}{}^{**}(s)+\text{Ø}_{03}{}^{**}(s)\text{Ø}_{3,12}{}^{**}(s)+\text{Ø}_{01}{}^{**}(s)\text{Ø}_{14}{}^{**}(s)+\text{Ø}_{01}{}^{**}(s)\text{Ø}_{15}{}^{**}(s)+$$

$$\text{Ø}_{01}{}^{**}(s)\text{Ø}_{16}{}^{**}(s)+\text{Ø}_{02}{}^{**}(s)\text{Ø}_{27}{}^{**}(s)+\text{Ø}_{02}{}^{**}(s)\text{Ø}_{28}{}^{**}(s)+\text{Ø}_{02}{}^{**}(s)\text{Ø}_{29}{}^{**}(s)$$

$$D(s)=1-\text{Ø}_{01}{}^{**}(s)\text{Ø}_{10}{}^{**}(s)-\text{Ø}_{02}{}^{**}(s)\text{Ø}_{20}{}^{**}(s)-\text{Ø}_{03}{}^{**}(s)\text{Ø}_{30}{}^{**}(s)$$

where $\text{N}=8907.480982, D=0.6391011951$

Mean time to unit/compressor MTSF =13937.512 hrs

Availability Analysis

Let A_i (t) be the probability that the system is in upstate at instant t given that the system entered regenerative state i at t=0. In steady state availability of the system is given by

$$A_0=\lim_{s\to 0}(sA_0^*(s))=\text{N}_1/D_1$$

where

$$N_1=\mu_0 p_{10}p_{20}p_{30}+p_{01}\mu_1(1-g_{11}{}^*)p_{20}p_{30}+p_{02}\mu_2(1-g_{12}{}^*)p_{10}p_{30}+p_{03}\mu_3(1-g_{13}{}^*)p_{10}p_{20}$$

$$D_1=(\mu_1(1-g_{11}{}^*)-m_{10})p_{20}p_{30}+(\mu_2(1-g_{12}{}^*)-m_{20})p_{10}p_{30}+(\mu_3(1-g_{13}{}^*)-m_{30})p_{10}p_{20}+m_{01}p_{01}p_{20}p_{30}+p_{10}m_{01}p_{20}p_{30}-(\mu_2(1-g_{12}{}^*)-m_{20})p_{10}p_{30}p_{01}$$

$$-(\mu_3(1-g_{13}{}^*)-m_{30})(p_{10}p_{20}p_{01})+p_{10}m_{02}p_{20}p_{30}+p_{02}m_{20}p_{10}p_{30}-(\mu_1(1-g_{11}{}^*)-m_{10})(p_{20}p_{30}p_{02})-(\mu_3(1-g_{13}{}^*)-m_{30})p_{10}p_{20}p_{02}+p_{10}m_{03}p_{20}p_{30}$$

$$+p_{03}m_{30}p_{10}p_{20}-(\mu_1(1-g_{11}{}^*)-m_{10})p_{03}p_{30}p_{20}-(\mu_2(1-g_{12}{}^*)-m_{20})p_{10}p_{30}p_{03}$$

N_1 =115.3779634, D_1=115.3779738

Availability of the unit/compressor (A_0) =0.999999999

Particular Cases

For graphical representation, let us suppose that $g_{11}(t)=\alpha_{11}e^{-\alpha_{11}t}, g_{12}(t)=\alpha_{12}e^{-\alpha_{12}t}, g_{13}(t)=\alpha_{13}e^{-\alpha_{13}t}$

Using the above particular case, the following values are estimated as

$$\alpha_{11} = 0.006896, \alpha_{12} = 0.000586, \alpha_{13} = 0.04166, \alpha_{21} = 0.0000983, \alpha_{22} = 0.0001347, \alpha_{23} = 0.00015873, \lambda_{11}, \lambda_{12}, \lambda_{13} = 0.00003868, \lambda_{21}, \lambda_{22}, \lambda_{23} = 0.0007352$$

Graphical Interpretation

Graph represents the behaviour of MTSF (Model 1 and Model 2) and Availability (Model 2) with failure rate λ_{12} having variation in λ_{22}.It is clear that as failure rate λ_{12} increases MTSF (Model 1 and Model 2) and Availability (Model 2) decreases. As the variation is taken in failure rate λ_{22} for MTSF(Model 1 and Model 2) and Availability(Model 2) it can be concluded that as the failure rate λ_{22} increases MTSF (Model 1 and Model 2)and Availability(Model 2 decreases).

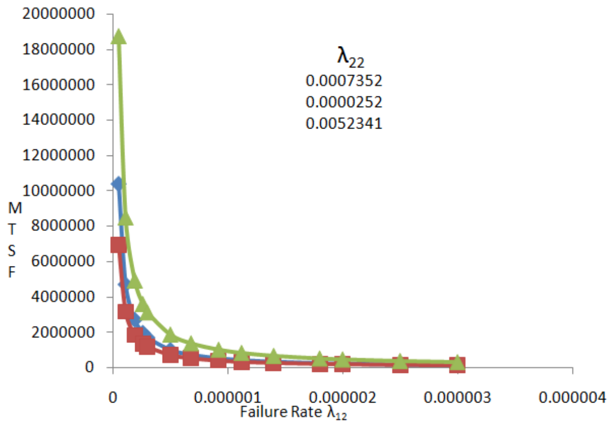
The measures of system effectiveness are obtained as:

Mean time to unit/compressor MTSF (Model 1 and Model 2) =13937.512 hrs.

Availability of the unit/compressor (A_0) (Model 1) =0.6156

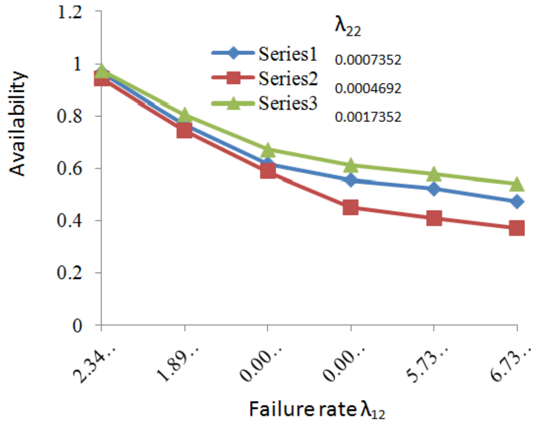
Availability of the unit/compressor (A_0) (Model 2) =0.999999999

For both the Models, the expected time for which the unit/compressor is in operation before it completely fails is about 13937.512 hours. It can be concluded that for the given system as the availability for Model 2 is greater than Model 1 in every situation hence priority for the failed unit is not recommended for present system.



Model 1 and Model 2

Figure 3: Graph between MTSF and λ12 with Variation in λ_{22}



Model 2

Figure 4: Graph between Availability and λ12 with Variation in λ_{22}

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